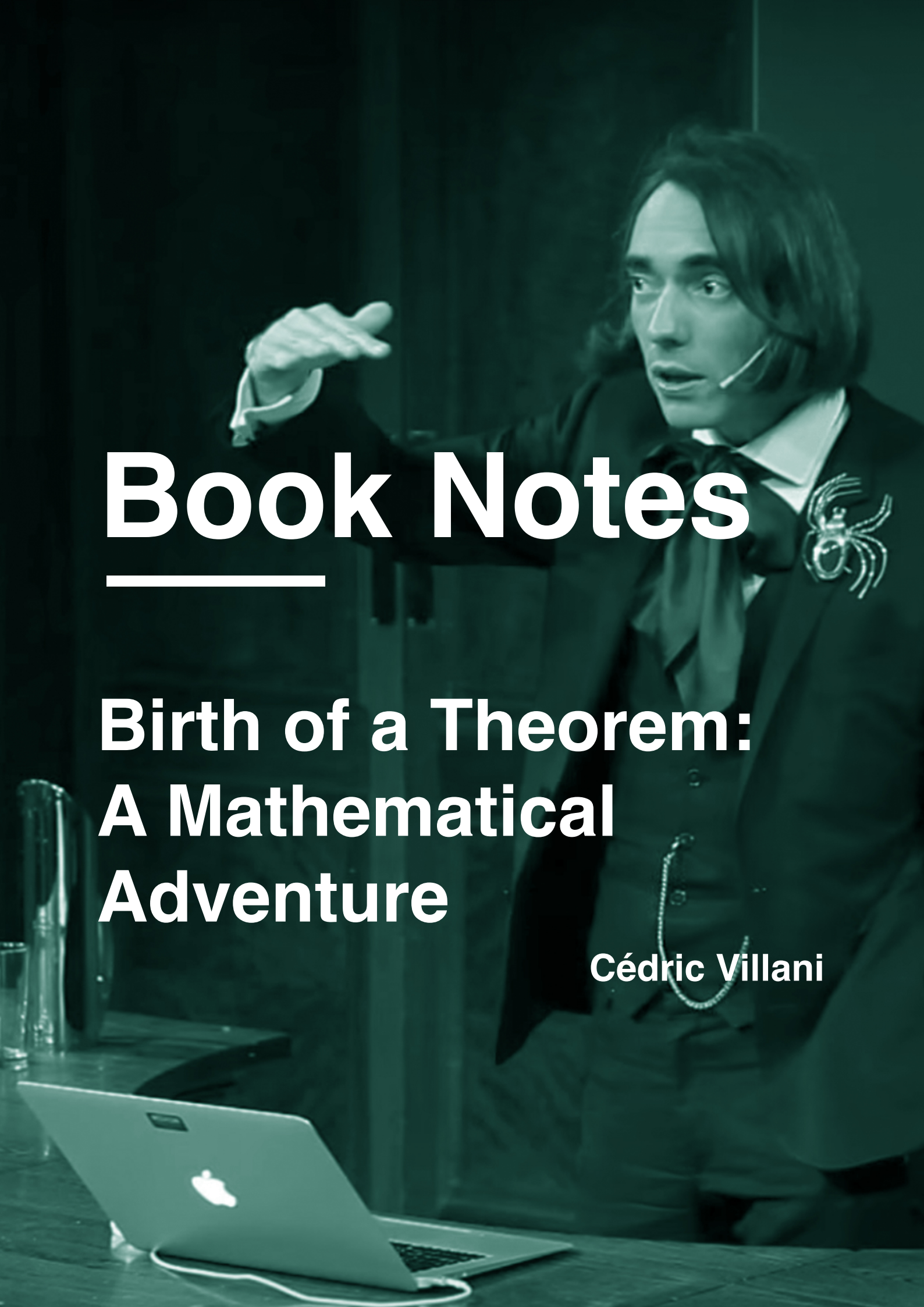

Book Notes

Birth of a Theorem: A Mathematical Adventure

Cédric Villani

Birth of a Theorem: A Mathematical Adventure by Cédric Villani

Average reading time - 5 minutes

John Nash, my mathematical hero, is said to have **regularly put himself under fantastic pressure by announcing results that he did not yet know how to prove.** On my way home that day I began to feel something like the pressure Nash must have felt. The terrible sense of urgency that now hung over me was not going to go away until I succeeded, one way or another, in completing this proof. I had to complete it— or else forever be disgraced!!

At first **Nash's questions were naïve, questions a beginner might ask.** Nirenberg started to wonder whether Nash really deserved his reputation. **It takes courage, when you are already famous and admired, to act the novice in a field you haven't yet mastered— courage, or else a quite unusual degree of self-confidence! The willingness to risk the embarrassment of appearing less than brilliant in the hope that the answer to an unexpected query will point you in the right direction.** But this is the price of making progress on a difficult problem. . . . **Little by little Nash's questions became more precise, more pertinent. An idea was beginning to form in his mind.**

I'd taken a risk, presenting completely new results that were still only half-baked, but the gamble paid off: **their criticisms enabled me to make much faster progress than I could have otherwise! Once again I had to put myself in a vulnerable position in order to become stronger.**

The ability to detect hidden connections between different areas of mathematics is what has made my reputation. These connections are invaluable! It's a bit like a game of Ping-Pong: every discovery you make on one side helps you discover something new on the other. The connections make it possible to see more of the landscape on both sides.

Doing mathematics is no different, really. You're constantly exploring, your eyes and ears are always open, and then every once in a while you're completely smitten by something and you pour your heart and soul into it, you tell yourself over and over again, hundreds and hundreds of times, that nothing else matters. Well, almost nothing else. **Sometimes the two worlds communicate with each other. Certain pieces of music that have kept me going in the course of a project are forever associated with moments of intense emotion.**

Once the song was finished **I played it again, and again, and again. I needed this loop, needed to go round and round, in order to leap forward. Work, Cédric, work.**

Each time, a personal encounter set everything in motion. It was as though I had acted as a catalyst! But I also firmly believe in the importance of searching for preexisting harmonies— after all, Newton, Kepler, and so many others have already shown us the way. **Everywhere you look, the world is filled with unsuspected connections!**

An article of thirty pages or so that doesn't resolve any major difficulty is unlikely to do the trick, however good it may be otherwise. **Deep down I am convinced that the solution will require completely new tools, which will allow us to look at the problem in a new way**. **I need a new norm. A norm, in mathematical jargon, is a special sort of ruler, or measuring stick, designed for the purpose of estimating the size of some quantity one wishes to investigate.**

That's what I love most of all about our small but very productive laboratory— the way conversation moves from one topic to another, especially when you're talking with someone whose mathematical interests are different from yours. **With no disciplinary barriers to get in the way, there are so many new paths to explore!**

Additionally, Boltzmann defined the equilibrium state of a statistical system as the state of maximum entropy, thus founding a vast field of research known as equilibrium statistical physics. In so doing, he demonstrated **that the most disordered state is the most natural state of all.**

Every mathematician worthy of the name **has experienced, if only rarely, the state of lucid exaltation in which one thought succeeds another as if miraculously. . . .** Unlike sexual pleasure, this feeling may last for hours at a time, even for days.

Donald Knuth is the living god of computer science. As one of my colleagues put it, "If Knuth were to walk into the hall in the middle of a lecture, everyone would bow down before him." Knuth took early retirement from Stanford and **turned of fhis email in order to devote himself full-time to completing his major work, The Art of Computer Programming, begun almost fifty years earlier. In the meantime, the three volumes already published by 1976 had revolutionized the subject.**

Tomorrow I'm taking care of the kids; there's no school on account of the snowstorm. But come Tuesday, the final push begins. **One way or another the Problem simply has got to be tamed, even if it means going without sleep. I'm going to take Landau with me everywhere— in the woods, on the beach, even to bed. Time now for him to watch out!**

and emphasized Cohen's insistence **on solving problems ex nihilo**, without relying on the work of others.

"**Cohen didn't believe in incremental mathematics." "Incremental?" "Yes, he thought that mathematics progresses by sudden leaps. You and me, like everyone else, nudge it forward by improving on the work of others, but not Cohen!** If you told him you were improving something he'd send you packing. He believed in revolutions and nothing else!"

Eric suggested that the effect was chimerical— a product of physicists' fertile imaginations that had no hope of being rigorously formulated in mathematical terms. None of this meant much to me at the time, **but I did manage to make a mental note and file it away in a corner of my brain.**

Paul Erdős, who helped found probabilistic number theory, was condemned to a life of restless wandering. Amazingly prolific, Erdős wrote some fifteen hundred articles (a world record), **roaming the length and breadth of the globe in his threadbare clothes, having neither home nor family nor job, only his suitcase, his notebook, and his genius.**

No one who is familiar with the world of science can have failed to notice how **many of the most gifted mathematicians and physicists of the twentieth century were Jews, or how many of the greatest geniuses were Hungarian** (many of them, but by no means all, Jews). Scientists who worked on the Manhattan Project in the 1940s were fond of saying that Martians really do exist: **they have superhuman intelligence, speak an incomprehensible language, and claim to come from a place called Hungary.**

But in order to solve a problem, you've got to know at the outset exactly what the problem is! In mathematical research, **clearly identifying what it is you are trying to do is a crucial, and often very tricky, first step.**

Grigori Perelman dumbfounded everyone by announcing a solution of Poincaré's conjecture. **A solution on which he had worked alone, in secret, for seven years!!**

On the obverse of the medal, Archimedes is shown in right-facing profile together with the inscription **TRANSIRE SUUM PECTUS MUNDOQUE POTIRI (Rise above oneself and grasp the world).** On the reverse, laurels frame an illustration of a theorem by Archimedes on the calculation of volumes of spheres and cylinders, with the inscription **CONGREGATI EX TOTO ORBE MATHEMATICI OB SCRIPTA INSIGNIA TRIBUERE (Mathematicians gathered from all over the world have paid tribute to a remarkable work).**
